Reflexive Neurodynamics

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Abstract

Behavioral dynamics of living species is characterized by additonal non-Newtonian properties which are not included in the laws of Newtonian or statistical mechanics. These properties follow from a priviledged ability of living systems to process information, namely, to generate a force in response to an external cue. In this paper "free-will-based" forces will be incorporated into the neurodynamical formalism.

1. Introduction

Recent advances in nonlinear dynamics demonstrate a remarkable complexity of patterns outside of equilibrium which are derived from simple basic laws of physics. There has been identified a class of mathematical models providing a variety of such patterns in the form of static, periodic or chaotic attractors. These models appeared to be so general that they predict not only physical, but also biological, economical and social patterns of behavior. Such a phenomenological reductionism may suggest that, on the dynamical level of description, there is no difference between a solar system, a swarm of insects, and a stock market. However, this conclusion is wrong for a very simple reason: even primitive living species possess additional non-Newtonian properties which are not included in the laws of Newtonian or statistical mechanics. These properties follow from a privileged ability of living species to process information, namely, to generate a force in response to an external cue. In this paper we will incorporate these "free-will-based" forces into the mathematical formalism of neurodynamics.

2. Dynamical Model

In contradistinction to physical systems, a biological system, from the viewpoint of nonlinear dynamics, can be considered as a multi-body system (with "bodies" represented by cells) which is interconnected via information flows. Since these flows as well as responses to them may be distorted, delayed, or incomplete, the motion of each cell becomes stochastic, and it can be simulated by a controlled random walk. This random walk is caused not by an external noise (as in the case of a physical particle) but rather by an internal effort (a "free will") triggered by the signaling system. Physically it is represented by an ordered sequence of runs, pausing and tumples.

One of the main challenges in modeling living systems is to distinguish a random walk of physical origin (for instance, Brownian motions) from those of biological origin and that will constitute the starting point of the proposed approach. As conjectured in [[1] the biological random walk must be nonlinear. Indeed, any stochastic Markov process can be described by linear Fokker-Planck equation (or its discretized version) only that types of processes has been observed in the inanimate world. However, all such processes always converge to a stable (ergodic or periodic) state, i.e., to the states of a lower complexity and higher entropy. At the same time, the evolution of living systems is directed toward a higher level of complexity if complexity is associated with a number of structural variations. The simplest way to mimic such a tendency is to incorporate a nonlinearity into the random walk; then the probability evolution will attain the features of the Burgers equation: the formation and dissipation of shock waves initiated by small shallow wave disturbances. As a result, the evolution never "dies": it produces new different configurations which are accompanied by increase or decrease of entropy (the decrease takes place during formation of shock waves, the increase-during their dissipation). In other words, the evolution can be directed "against the second law of thermodynamics by forming patterns outside of equilibrium.

In order to elucidate both the physical and the biological aspects of the proposed model, let us start with a one-dimensional random walk:

$$x_{t+\tau} = x_t + h\, Sgn[R(\pm 1) + \mu] \quad h = Const,\ \tau = Const, \tag{1}$$

where h and τ are the space (along x) and time steps respectively; $R(\pm 1)$ is a random function taking values from -1 to 1 with equal probability, μ is a control parameter while $|\mu| \leq 1/2$. (Physical implementations of this model were discussed [1])

Eq. (1) describes motion in actual physical space. But since this motion is irregular, it is more convenient to turn to the probability space:

$$f_{t+\tau} = pf_{x-h} + (1-p)f_{x+h}, \quad f_{t+\tau} = f(x, t+\tau), \text{ etc.} \tag{2}$$

where $f(x,t)$ is the probability that the moving particle occupies the point x at the instant t, and the transition probability

$$p = \frac{1}{2} + \mu, 0 \leq p \leq 1 \tag{3}$$

It is well known that if the system interacts with the external world, i.e.,

$$\mu = \mu(x), \text{ and therefore, } p = p(x)$$

then the solution to Eq. (2) subject to the reflecting boundary conditions converges to a stable stochastic attractor. However, if

$$\mu = \mu(f), \text{ and therefore, } p = p(f), \tag{4}$$

Eq. (2) becomes nonlinear, and Eq. (1) is coupled with Eq. (2) via the feedback (4).

From the physical viewpoint, the system (1), (2) can be compared with the Langevin equation which is coupled with the corresponding Fokker-Planck equation such that the stochastic force is fully defined by the current probability distributions, while the diffusion coefficient is fully defined by the stochastic force. The process described by this system is Markovian since future still depends only upon present, but not past. However, now present includes not only values of the state variable, but also its probability distribution, and that leads to nonlinear evolution of random walk.

From the mathematical viewpoint, Eq. (1) simulates probabilities while Eq. (2) manipulates by their values. The connection between these equations is the following: if Eq. (1) is run independently many times and a statistical analysis of these solutions is performed, then the calculated probability will evolve according to Eq. (2).

From the biological viewpoint, Eqs. (1) and (2) represent the same subject: a living specie. Eq. (1) simulates its motor dynamics, i.e., actual motion in physical space, while Eq. (2) can be associated with mental dynamics describing information flows in the probability space.

Such an interpretation [1] was evoked by the concept of reflection in psychology. Reflection is traditionally understood as the human ability to take the position of an observer in relation to one's own thoughts. In other words, the reflection is a self-awareness via the interaction with the "image of the self." In terms of the phenomenological formalism proposed above, Eq. (2) represents the probabilistic "image" of the dynamical system (1). If this system "possesses" its own image, then it can predict, for instance, future expected values of its parameters, and, by interacting with the image, change the expectations if they are not consistent with the objective. In this context, Eq (1) simulates acting, and Eq (2) simulates "thinking." Their interaction can be implemented by incorporating probabilities, its functions and functionals into the control parameter μ (see Eq. (4)).

3. Emerging Self-organization

We will start the analysis of the coupled motor-mental dynamics with Eqs. (1) and (2) where

$$p = Sin^2(\alpha f + \beta), \mu = p - \frac{1}{2}, \alpha, \beta = Const, \; f = f(x,t) \tag{5}$$

i.e.

$$x_{t+\tau} = x_t + h\, Sgn\left[R(\pm 1) + Sin^2(\alpha f + \beta) + \frac{1}{2} \right] \tag{6}$$

$$f_{t+\tau} = f_{x-h}\, Sin^2(\alpha f + \beta) + f_{x+h} Cos^2(\alpha f + \beta) \tag{7}$$

Here α and β are constant weights

In order to illustrate the fundamental nonlinear effects, we will analyze the behavior of special critical points by assuming that

$$\alpha = \frac{5\pi}{12}, \; \beta = -\frac{\pi}{6} \; \text{and}$$

$$f_0 = f(t=0) = \begin{cases} f_0^{(1)} = \dfrac{1}{5} \; at \; x = -\ell \\ f_0^{(2)} = \dfrac{4}{5} \; at \; x = \ell \\ f_0^{(3)} = 0 \; \text{otherwise} \end{cases} \tag{8}$$

Then the solution to Eq. (7) will consist of two waves starting from the points $x = -\ell$ and $x = \ell$, traveling toward each other with the constant speed $\upsilon = h/\tau$, and transporting the values $f_0^{(1)}$ and $f_0^{(2)}$, respectively, i.e.,

$$f = f_0^{(1)}\left(-\ell + \frac{h}{\tau}n\right) + f_0^{(2)}\left(\ell - \frac{h}{\tau}n\right), \; n = 0, 1, \ldots \frac{\ell}{h}, \tag{9)}$$

where n is the number of time-steps.

At $n = 1/h$, the waves confluence into one solitary wave at $x = 0$:

$$f = \begin{cases} 1 \text{ at } \; x = 0 \\ 0 \text{ otherwise} \end{cases} \qquad \text{at } \; t = n\tau = \frac{\ell}{h}\tau \tag{10}$$

This process represents a discrete version of formation and confluence of shock waves, and it is characterized by a decrease of the Shannon entropy from

$$H(0) = -\frac{1}{5}\ell og_2\frac{1}{5} - \frac{4}{5}\ell og_2\frac{4}{5} > 0 \; to \; H(n\tau) = 0 \tag{11}$$

However, the solitary wave (10) is not stationary. Indeed, as follows from Eq. (7), the solution at $t = (n+1)\tau$ splits into two equal values:

$$f_{(n+1)\tau} = \begin{cases} 1/2 \; at \; x = \ell \pm h \\ 0 \quad otherwise \end{cases} \tag{12}$$

The process (12) can be identified as a discrete version of diffusion during which the entropy increases again from

$$H(n\tau) = 0 \;\; to \;\; H[(n+1)\tau] = -\ell og_2\frac{1}{2} = 1 \tag{13}$$

The further evolutionary steps $t \geq (n+2)\tau$ will include both diffusion and wave effects, and therefore, the solution will endlessly display more and more sophisticated patterns of behavior in the probability space. The corresponding solutions to Eq. (6) represent samples of the stochastic process (7) in the form of non-linear random walks in actual physical space.

Thus, the solutions to coupled motor-mental dynamics simulate emerging self-organization which can start spontaneously. At this level of description, such an effect is triggered by instability rather than by a specific objective. In other words, the model represents a "brainless" life. However, it serves well to the global objective of each living system: the survival. Indeed, it is a well established fact in biology that marginal instability makes behaviors of living system more flexible and therefore, more adaptable to changing environment.

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References

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